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SCATTERING OF SURFACE WAVES BY THE EDGE OF A FLOATING ELASTIC PLATE

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The diffraction of plane surface waves by a semi-infinite floating plate in a fluid of finite depth is studied. An explicit analytical solution of the problem is obtained using the Wiener–Hopf technique. Simple exact formulas for reflection and transmission coefficients and their asymptotic expressions are derived. Results of numerical calculations using the obtained formulas are presented.

The behavior of a floating flexible plate in waves has been examined previously in studies of flexural-gravity waves in a liquid covered by an ice sheet (see reviews [1, 2]). Recent interest in this problem is motivated by its relation to the design of artificial islands, floating airports and various marine platforms. Various numerical methods have been developed to solve the problem, in particular, for an infinite plate (see, e.g., [3–5]). However, all these methods yield reliable results only for large and intermediate wavelengths and are inadequate for short incident waves. Analytical solutions based on the Wiener–Hopf technique were obtained for oblique incidence in a fluid of finite depth [6], for a stratified fluid [7], for normal incidence in an infinitely deep fluid [8], and for oblique incidence using the Timoshenko–Mindlin equation for flexural vibrations of a plate [9]. The solutions derived in all the papers cited are expressed in terms of unknown coefficients which are determined from a system of linear algebraic equations. The coefficients of the system have a complicated form. The explicit solution of the system obtained in [10] yields exact values of the sought-for coefficients, an explicit expression for the velocity potential, and simple exact formulas for reflection and transmission coefficients in the case of normal incidence in an infinitely deep fluid. In the present paper, similar formulas are derived for a fluid of finite depth.

Formulation of the Problem. We assume that the surface of an ideal and incompressible fluid of finite depth H is partly covered by a semi-infinite thin elastic plate. A plane incident wave of small amplitude propagates normally to the edge of the plate, and the length of the incident wave is great compared to the plate thickness. We introduce Cartesian coordinates (x, y) with origin O at the plate edge and the Ox axis directed along the plate (Fig. 1). The draught of the plate is neglected so that the boundary conditions are imposed at the undisturbed free-surface level. The problem is solved in a linear formulation.

The velocity potential φ satisfies the Laplace equation

$$\Delta\varphi = 0 \qquad (y < 0). \tag{1}$$

The boundary conditions are written as

$$\frac{\partial\varphi}{\partial y} = 0 \quad (y = -H, \quad -\infty < x < \infty), \qquad \varphi_y = \eta_t \quad (y = 0, \quad -\infty < x < \infty),$$

$$D\,\frac{\partial^4\eta}{\partial x^4} + \rho_0 h\,\frac{\partial^2\eta}{\partial t^2} = p, \qquad p = -\rho(\varphi_t + g\eta) \qquad (y = 0, \quad x > 0), \tag{2}$$

$$\varphi_t + g\eta = 0 \qquad (y = 0, \quad x < 0).$$

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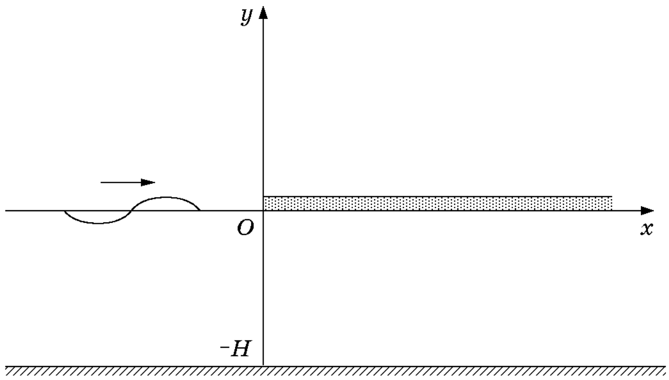

Fig. 1. Scheme of the flow.

Here η is the vertical displacement of the free surface (plate), g is the acceleration of gravity, D is the flexural rigidity, h and ρ_0 are thickness and density of the plate, respectively, and t is time. At the plate edge, the bending moment and the external force must be zero:

$$\frac{\partial^2\eta}{\partial x^2} = \frac{\partial^3\eta}{\partial x^3} = 0 \qquad (x = 0, \quad y = 0). \tag{3}$$

We introduce the nondimensional variables $\varphi' = \varphi/(A\sqrt{gl}\,)$, $x' = x/l$, $y' = y/l$, $t' = t\sqrt{g/l}$, and $H' = H/l$, where A is the incident-wave amplitude and $l = g/\omega^2$ is the characteristic length. Below, the primes are omitted. The potential φ can be expressed as

$$\varphi = (\varphi_0 + \varphi_1)\mathrm{e}^{-i\omega t}, \qquad \varphi_0 = \mathrm{e}^{i\gamma x}\cosh(\gamma(y + H))/\cosh(\gamma H),$$

where φ_0 is the incident-wave potential, φ_1 is the diffraction potential, γ is the wavenumber of the incident wave, defined by the dispersion relation for surface waves at water depth H: $\gamma\tanh(\gamma H) - 1 = 0$. Then, using (1)–(3), we can formulate a boundary-value problem for φ_1. The function $\varphi_1(x, y)$ satisfies Eq. (1) and the following boundary conditions:

$$\frac{\partial\varphi_1}{\partial y} = 0 \qquad (y = -H, -\infty < x < \infty); \tag{4}$$

$$\frac{\partial\varphi_1}{\partial y} - \varphi_1 = 0 \qquad (y = 0, \quad x < 0); \tag{5}$$

$$\Big(\beta\,\frac{\partial^4}{\partial x^4} + 1 - \delta\Big)\frac{\partial\varphi_1}{\partial y} - \varphi_1 = B\,\mathrm{e}^{i\gamma x} \qquad (y = 0, \quad x > 0); \tag{6}$$

$$\frac{\partial^2}{\partial x^2}\,\frac{\partial\varphi}{\partial y} = \frac{\partial^3}{\partial x^3}\,\frac{\partial\varphi}{\partial y} = 0 \qquad (x = 0, \quad y = 0). \tag{7}$$

Here $\beta = D/(\rho g l^4)$, $\delta = \rho_0 h/(\rho l)$ are nondimensional parameters of the problem and $B = \delta - \beta\gamma^4$. In addition, radiation conditions should be satisfied for $|x| \to \infty$ and the solution should be regular at the plate edge, i.e., the energy must be limited locally. The above assumptions imply that $\delta \ll 1$. Here and below, the value of δ is set equal to zero.

Dispersion Relations. Let us consider the propagation of waves in a fluid with a free surface and under the plate. The corresponding solutions of the Laplace equation should satisfy condition (4) at the bottom and the pertinent condition at the upper boundary, written as $\mathrm{e}^{i\alpha x}\cosh(\alpha(y + H))/\cosh(\alpha H)$.

1. *Surface Waves.* For surface waves, the values of α must satisfy the dispersion relation $\alpha\tanh(\alpha H) - 1 = 0$, which has two real roots $\pm\gamma$ and a countable set of purely imaginary roots $\pm\gamma_j$ $(j = 1, 2, \ldots)$, which are symmetric about the real axis [3].

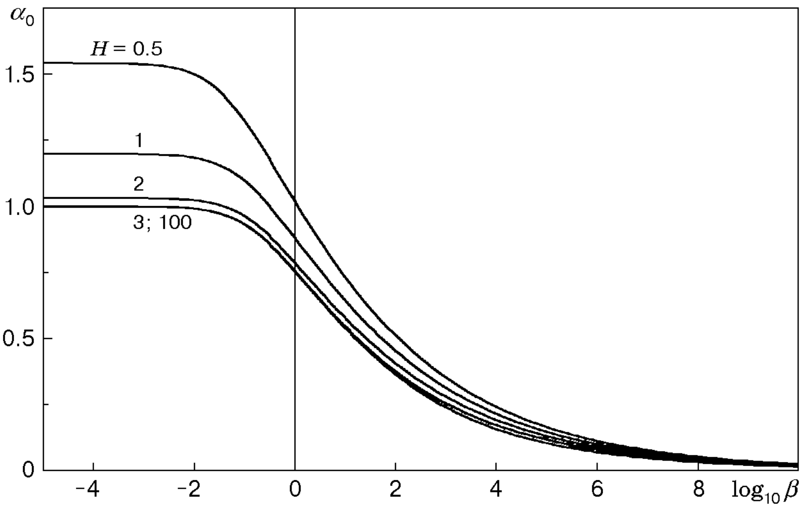

Fig. 2. Wavenumber α_0 versus β for various H.

2. *Flexural-Gravity Waves.* For waves propagating in the plate (so-called flexural-gravity waves), the dispersion relation $(\beta\alpha^4 + 1)\alpha\tanh(\alpha H) - 1 = 0$ has two real roots $\pm\alpha_0$, a countable set of purely imaginary roots $\pm\alpha_j$ $(j = 1, 2, \ldots)$, which are symmetric about the real axis, and four complex roots, which are symmetric about the real and imaginary axes [3]. The roots located in the first and second quadrants are denoted by α_{-1} and α_{-2}, respectively.

The real roots of the dispersion relations correspond to propagating waves while the remaining roots define edge waves, which decay exponentially away from the edge. The dependence of the wavenumber α_0 on β for various values of H is shown in Fig. 2. For $H \geqslant 3$, the values of α_0 are practically independent of H. However, for small values of H and β, the effect of water depth becomes significant, so that $\alpha_0 \to \infty$ as $H \to 0$. For the limiting values of H and β, the asymptotic behavior of the root α_0 is given by the following relations: $\alpha_0 \to (\beta H)^{-1/6}$ as $H \to 0$; at the limit $H \to \infty$, the value of α_0 tends to the real root of the equation $\beta\alpha^5 + \alpha - 1 = 0$, which corresponds to an infinitely deep fluid [10]. In addition, $\alpha_0 \to (\beta H)^{-1/6}$ as $\beta \to \infty$ and $\alpha_0 \to \gamma$ as $\beta \to 0$.

Analytical Solution of the Problem. The solution is derived using Jones' version of the Wiener–Hopf technique [11]. We introduce the following functions of the complex-valued variable α:

$$\Phi_+(\alpha, y) = \int_0^\infty e^{i\alpha x}\varphi_1(x, y)\,dx, \qquad \Phi_-(\alpha, y) = \int_{-\infty}^0 e^{i\alpha x}\varphi_1(x, y)\,dx, \tag{8}$$

$$\Phi(\alpha, y) = \Phi_+(\alpha, y) + \Phi_-(\alpha, y).$$

The functions $\Phi_+(\alpha, y)$ and $\Phi_-(\alpha, y)$ are defined in the upper $(\operatorname{Im}\alpha > 0)$ and lower $(\operatorname{Im}\alpha < 0)$ half-planes, respectively. By analytic continuation, these functions can be defined over the entire complex plane.

Let us study the behavior of the functions $\Phi_\pm(\alpha, y)$. At the limit $x \to -\infty$, the diffraction potential represents a reflected wave of the form $R\,e^{-i\gamma x}$ and a set of exponentially decaying waves. The wave that decays most slowly corresponds to the root γ_1. Hence, $\Phi_-(\alpha, y)$ is analytic in the half-plane $\operatorname{Im}\alpha < |\gamma_1|$ except in the pole at $\alpha = \gamma$. At the limit $x \to \infty$, the potential φ_1 represents a transmitted wave with wavenumber α_0, a wave with wavenumber γ, which compensates for φ_0, and a set of exponentially decaying modes. Therefore, the function $\Phi_+(\alpha, y)$ is analytical in the half-plane $\operatorname{Im}\alpha > -c$ except in the poles at $\alpha = -\alpha_0$ and $\alpha = -\gamma$ ($c = \min\{\operatorname{Im}\alpha_{-1}, |\alpha_1|\}$ is a positive number that corresponds to the least decaying mode in the plate).

The function $\Phi(\alpha, y)$ represents the Fourier transform of $\varphi_1(x, y)$ and satisfies the equation $\partial^2\Phi/\partial y^2 - \alpha^2\Phi = 0$. The general solution of this equation subject to condition (4) at the bottom has the form

$$\Phi(\alpha, y) = C(\alpha)\cosh(\alpha(y + H))/\cosh(\alpha H). \tag{9}$$

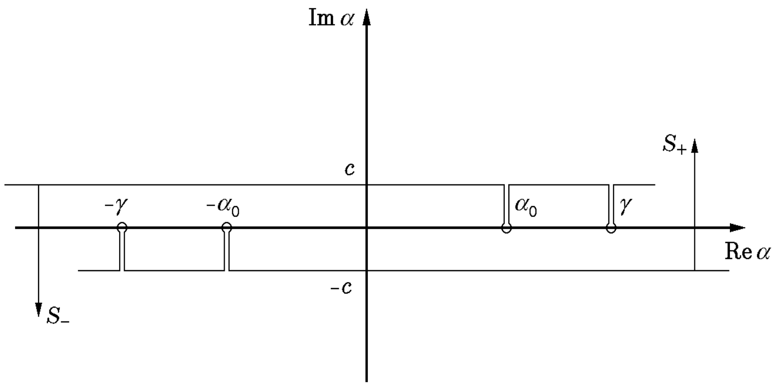

Fig. 3. Analyticity regions $S_\pm$ for the functions $\Phi_\pm$.

We use $D_\pm(\alpha)$ to denote integrals of the type (8) in which the integrand φ_1 is replaced by the left side of boundary condition (5); $F_\pm(\alpha)$ denotes similar integrals with the integrand given by the left side of expression (6). These integrals are Fourier transforms of the generalized functions [12] and satisfy the following relations:

$$D_+(\alpha) + D_-(\alpha) = C(\alpha)(\alpha \tanh(\alpha H) - 1), \quad F_+(\alpha) + F_-(\alpha) = C(\alpha)[(\beta\alpha^4 + 1)\alpha \tanh(\alpha H) - 1]. \tag{10}$$

From boundary conditions (5) and (6), we have $D_-(\alpha) = 0$ and $F_+(\alpha) = -B/(i(\alpha+\gamma))$. In view of this, relations (10) are written as

$$D_+(\alpha) = C(\alpha)(\alpha \tanh(\alpha H) - 1), \quad F_-(\alpha) - \frac{B}{i(\alpha+\gamma)} = C(\alpha)[(\beta\alpha^4 + 1)\alpha \tanh(\alpha H) - 1]. \tag{11}$$

Equations (11) yield

$$D_+(\alpha) = \frac{\alpha \tanh(\alpha H) - 1}{(\beta\alpha^4 + 1)\alpha \tanh(\alpha H) - 1}\Big(F_-(\alpha) - \frac{B}{i(\alpha+\gamma)}\Big). \tag{12}$$

Let us introduce $K(\alpha) = K_1(\alpha)/K_2(\alpha)$, where $K_1(\alpha)$ and $K_2(\alpha)$ are dispersion functions for the free-surface waves and the flexural-gravity waves, respectively: $K_1(\alpha) = \alpha \tanh(\alpha H) - 1$ and $K_2(\alpha) = (\beta\alpha^4+1)\alpha \tanh(\alpha H) - 1$. It should be noted that these functions are even.

Following the Wiener–Hopf technique, we must factorize the function $K(\alpha)$, i.e., write it as

$$K(\alpha) = K_+(\alpha)K_-(\alpha), \tag{13}$$

where the functions $K_\pm(\alpha)$ are regular in the same domains as the functions $\Phi_\pm(\alpha, y)$. The function $K(\alpha)$ has zeros and poles on the real axis at the points $\pm\gamma$ and $\pm\alpha_0$, respectively. It can easily be shown that $|\alpha_1| < |\gamma_1|$. Therefore, we consider the analyticity regions S_+ and S_- (S_+ is the half-plane $\operatorname{Im}\alpha > -c$ with cuts that exclude the points α_0 and γ, S_- is the half-plane $\operatorname{Im}\alpha < c$ with cuts that exclude the points $-\alpha_0$ and $-\gamma$) (see Fig. 3).

Let us introduce the function $g(\alpha) = K(\alpha)\beta(\alpha^2-\alpha_0^2)(\alpha^2-\alpha_{-1}^2)(\alpha^2-\alpha_{-2}^2)/(\alpha^2-\gamma^2)$. The function $g(\alpha)$ has no zeros, is bounded, and tends to unity at infinity. We factorize $g(\alpha)$ as follows [11]: $g(\alpha) = g_+(\alpha)g_-(\alpha)$, where

$$g_\pm(\alpha) = \exp\left[\pm\frac{1}{2\pi i}\int_{-\infty\mp id}^{\infty\mp id} \frac{\log g(x)}{x-\alpha}\,dx\right], \qquad d < c. \tag{14}$$

The functions $K_\pm(\alpha)$ are defined by

$$K_+(\alpha) = \frac{(\alpha+\gamma)g_+(\alpha)}{\sqrt{\beta}(\alpha+\alpha_0)(\alpha+\alpha_{-1})(\alpha+\alpha_{-2})}, \quad K_-(\alpha) = \frac{(\alpha-\gamma)g_-(\alpha)}{\sqrt{\beta}(\alpha-\alpha_0)(\alpha-\alpha_{-1})(\alpha-\alpha_{-2})}, \tag{15}$$

and $K_+(\alpha) = K_-(-\alpha)$.

Equation (12) is written as

$$K_-(\alpha)\Big(F_-(\alpha) - \frac{B}{i(\alpha+\gamma)}\Big) = \frac{D_+(\alpha)}{K_+(\alpha)}$$

or

$$K_-(\alpha)F_-(\alpha) - \frac{B}{i(\alpha+\gamma)}\,(K_-(\alpha) - K_-(-\gamma)) = \frac{D_+(\alpha)}{K_+(\alpha)} + \frac{BK_-(-\gamma)}{i(\alpha+\gamma)}.$$

The functions on the left and right sides of this equation are analytical in the regions S_- and S_+, respectively. Analytic continuation of these functions defines a function that is analytical over the entire complex plane. By Liouville's theorem, this function is a polynomial. The degree of the polynomial is determined by the behavior of the functions as $|\alpha| \to \infty$.

Local limitedness of energy implies that the singularity at the plate edge is of order not higher than $O(r^{-\lambda})$ ($\lambda < 1$ and r is the distance from the plate edge). Hence, as $|\alpha| \to \infty$, the functions $F_-(\alpha)$ and $D_+(\alpha)$ have orders not less than $O(|\alpha|^{\lambda+3})$ and $O(|\alpha|^{\lambda-1})$, respectively [12]. Since $g_\pm(\alpha) \to 1$ at $|\alpha| \to \infty$, the functions $K_\pm(\alpha)$ are of order $O(|\alpha|^{-2})$ at infinity. Thus, the degree of the polynomial is equal to unity and

$$\frac{D_+(\alpha)}{K_+(\alpha)} + \frac{BK_-(-\gamma)}{i(\alpha+\gamma)} = \frac{BK_-(-\gamma)}{i}\,(a + b\alpha),$$

where a and b are unknown constants to be determined from (7).

Solving the last equation for $D_+(\alpha)$ and taking into account (9) and (11), we obtain

$$\varphi_1(x,y) = \frac{BK_-(-\gamma)}{2\pi i}\int_{-\infty}^{\infty} \mathrm{e}^{-i\alpha x}\,\frac{\cosh(\alpha(y+H))K_+(\alpha)}{\cosh(\alpha H)K_1(\alpha)}\Big(a + b\alpha - \frac{1}{\alpha+\gamma}\Big)d\alpha,$$

$$\frac{\partial\varphi}{\partial y}(x,0) = \mathrm{e}^{i\gamma x} + \frac{BK_-(-\gamma)}{2\pi i}\int_{-\infty}^{\infty} \mathrm{e}^{-i\alpha x}\,\frac{\alpha\tanh(\alpha H)K_+(\alpha)}{K_1(\alpha)}\Big(a + b\alpha - \frac{1}{\alpha+\gamma}\Big)d\alpha. \qquad (16)$$

The integration contour should completely lie in the domain of intersection of the regions S_+ and S_-. For example, the integration contour can run along the real axis, passing below the points α_0 and γ and above the points $-\alpha_0$ and $-\gamma$.

Let us consider the case $x > 0$. In view (13), the second expression in (16) is written as

$$\frac{\partial\varphi}{\partial y}(x,0) = \mathrm{e}^{i\gamma x} + \frac{BK_-(-\gamma)}{2\pi i}\int_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha x}\,\alpha\tanh(\alpha H)}{K_-(\alpha)K_2(\alpha)}\Big(a + b\alpha - \frac{1}{\alpha+\gamma}\Big)d\alpha.$$

For $x > 0$, we close the contour in the lower half-plane and obtain the poles at points $-\gamma$ and $-\alpha_j$ ($j = -2, -1, \ldots$). The residue at the point $-\gamma$ is compensated for by the incident wave. Hence,

$$\frac{\partial\varphi}{\partial y}(x,0) = -BK_-(-\gamma)\sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x}\,\alpha_j\tanh(\alpha_j H)}{K_-(-\alpha_j)K_2'(-\alpha_j)}\Big(a - b\alpha_j - \frac{1}{\gamma-\alpha_j}\Big).$$

Substituting this expression into boundary conditions (7), we obtain the following system of second-order algebraic linear equations for the unknowns a and b:

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{23} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}. \qquad (17)$$

The matrix coefficients are given by

$$A_{11} = \sum_{j=-2}^{\infty} \frac{\alpha_j^3\tanh(\alpha_j H)}{K_-(-\alpha_j)K_2'(-\alpha_j)}, \qquad A_{12} = iA_{21},$$

$$A_{21} = \sum_{j=-2}^{\infty} \frac{\alpha_j^4\tanh(\alpha_j H)}{K_-(-\alpha_j)K_2'(-\alpha_j)}, \qquad A_{22} = -\sum_{j=-2}^{\infty} \frac{\alpha_j^5\tanh(\alpha_j H)}{K_-(-\alpha_j)K_2'(-\alpha_j)}.$$

The quantities C_1 and C_2 entering into the right side of (17) have the forms

$$C_1 = \sum_{j=-2}^{\infty} \frac{\alpha_j^3 \tanh(\alpha_j H)}{K_-(-\alpha_j)K_2'(-\alpha_j)(\gamma - \alpha_j)}, \quad C_2 = \sum_{j=-2}^{\infty} \frac{\alpha_j^4 \tanh(\alpha_j H)}{K_-(-\alpha_j)K_2'(-\alpha_j)(\gamma - \alpha_j)}.$$

System (17) can be written as

$$\begin{pmatrix} \gamma U_3 - U_4 & -\gamma U_4 + U_5 \\ \gamma U_4 - U_5 & -\gamma U_5 + U_6 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} U_3 \\ U_4 \end{pmatrix}, \tag{18}$$

where $U_m = \sum\limits_{j=-2}^{\infty} \dfrac{\alpha_j^m \tanh(\alpha_j H)}{K_-(-\alpha_j)K_2'(-\alpha_j)(\gamma - \alpha_j)}$.

Let us show that the coefficients U_5 and U_6 are zero. Then, the system obtained can be solved explicitly. Replacing the sum by integral and taking into account that $\alpha_j^5 \tanh(\alpha_j H) = -K_1(\alpha_j)/\beta$, we obtain

$$U_m = \frac{(-1)^{m-1}}{2\pi i\beta} \int_{-\infty}^{\infty} \frac{K_1(\alpha)\alpha^{m-5}\,d\alpha}{(\gamma+\alpha)K_-(\alpha)K_2(\alpha)}, \qquad m = 5,\ 6.$$

In view of (13), we have

$$U_m = \frac{(-1)^{m-1}}{2\pi i\beta} \int_{-\infty}^{\infty} \frac{\alpha^{m-5}K_+(\alpha)\,d\alpha}{\gamma+\alpha}, \qquad m = 5,\ 6.$$

In the upper half-plane, the integrand is regular, and as $|\alpha| \to \infty$, it decreases not more slowly than α^{-2} does. Hence, $U_5 = U_6 = 0$. System (18) yields $a = 1/\gamma$ and $b = -1/\gamma^2$.

Substituting the expressions for a and b into (16), we obtain

$$\varphi_1(x,0) = -\frac{BK_-(-\gamma)}{2\pi i\gamma^2} \int_{-\infty}^{\infty} \mathrm{e}^{-i\alpha x} \frac{K_+(\alpha)\alpha^2\,d\alpha}{K_1(\alpha)(\gamma+\alpha)}.$$

We now can find the reflected and transmitted waves. For $|x| \to \infty$, the potential is given by $\varphi(x,0) = R\,\mathrm{e}^{-i\gamma x}$ as $x \to -\infty$ or $\varphi(x,0) = T\,\mathrm{e}^{i\alpha_0 x}$ as $x \to \infty$. The expressions for R and T are

$$R = \frac{\beta\gamma K_+^2(\gamma)}{2K_1'(\gamma)}, \qquad T = -\frac{\beta\gamma^2 K_-(-\gamma)\alpha_0^2}{(\gamma-\alpha_0)K_-(-\alpha_0)K_2'(-\alpha_0)}.$$

With allowance for the nondimensionalizing performed, the amplitude $|R|$ represents the reflection coefficient. Substituting expressions (15) into the above formulas, we obtain the amplitudes $|R|$ and $|T|$. The integral in Eq. (14) can be calculated along the real axis [11]. As a result, we obtain

$$|g_+(\gamma)| = \sqrt{g(\gamma)} = \sqrt{\frac{\beta(\gamma^2-\alpha_0^2)|\gamma^2-\alpha_{-1}^2|\,|\gamma^2-\alpha_{-2}^2|\,|K_1'(\gamma)|}{2\gamma|K_2(\gamma)|}},$$

$$|K_+(\gamma)| = \sqrt{\frac{2\gamma(\gamma-\alpha_0)|K_1'(\gamma)|}{(\gamma+\alpha_0)|K_2(\gamma)|}}.$$

The reflection coefficient is given by $|R| = |(\gamma-\alpha_0)/(\gamma+\alpha_0)|$. Similarly, we find

$$|K_+(\alpha_0)| = \sqrt{\frac{(\gamma+\alpha_0)|K_1(\alpha_0)|}{2\alpha_0(\gamma-\alpha_0)|K_2'(\alpha_0)|}}.$$

Then, the formula for $|T|$ becomes

$$|T| = \frac{2}{\gamma+\alpha_0}\sqrt{\frac{\gamma|K_1'(\gamma)|}{\tanh(\alpha_0 H)|K_2'(\alpha_0)|}}.$$

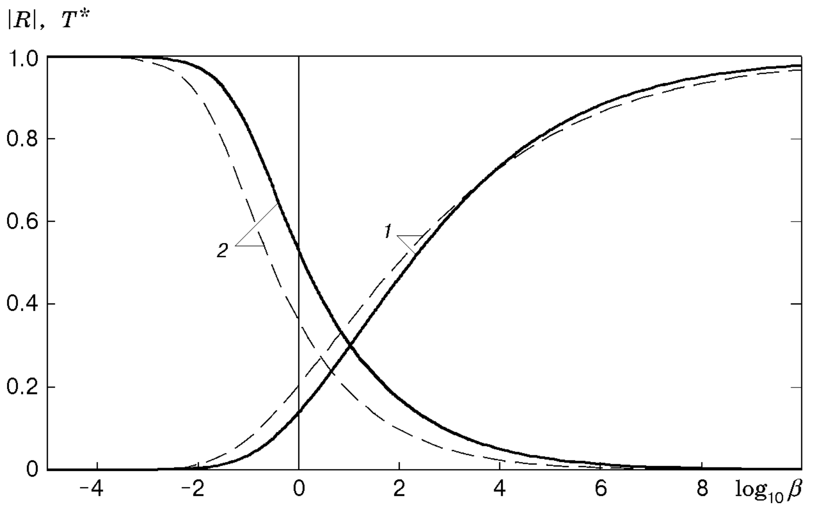

Fig. 4. Reflection coefficient $|R|$ (curves 1) and transmission coefficient T^* (curves 2) versus β for $H = 100$ (solid curves) and 0.5 (dashed curves).

For normal wave incidence, the exact energy-balance relation between $|R|$ and $|T|$ obtained in [6, 9] can be written as

$$|R|^2 + |T|^2 \frac{\alpha_0 \tanh(\alpha_0 H)|K_2'(\alpha_0)|}{|K_1'(\gamma)|} = 1.$$

This relation is satisfied by the expressions obtained for the amplitudes. We note that the exact formulas for $|R|$ and $|T|$ obtained in the present paper coincide with the approximate formulas given in [9], where $|R|$ is derived under the assumption of continuity of displacements across the edge, i.e., for $a = b = 0$, and the formula for $|T|$ is obtained from the energy-balance relation. Actually, the coefficients a and b are nonzero, but $a + b\gamma = 0$, which explains the good agreement between calculations by these formulas and calculations taking into account the parameter δ [9].

Next, we can evaluate the parameters of interest for the transmitted wave. The amplitude of plate displacements in the transmitted wave is given by $|\eta| = |T|\alpha_0 \tanh(\alpha_0 H)$. The right side of this equation represents the transmission coefficient

$$T^* = \frac{2\alpha_0}{\gamma + \alpha_0} \sqrt{\frac{\gamma |K_1'(\gamma)| \tanh(\alpha_0 H)}{|K_2'(\alpha_0)|}}.$$

In the literature, one can find various definitions of the transmission coefficient. In most papers, the transmission coefficient is defined as the ratio of the amplitude of vertical displacements in the transmitted wave to the incident-wave amplitude [3–6]. However, in [9], the transmission coefficient is defined as the ratio between the potentials amplitudes in transmitted and incident waves.

The plate deflection is given by

$$e_{xx} = -\frac{h}{2} \frac{\partial^2 \eta}{\partial y^2}.$$

The maximum deflection of the plate in the transmitted wave has the amplitude $e_{\max} = Ahe/(2l^2)$, where $e = |T|\alpha_0^3 \tanh(\alpha_0 H)$.

Limiting Cases. Let us find asymptotic formulas for the reflection and transmission coefficients for large and small values of H and β. For $\beta \to \infty$, i.e., for a very rigid plate or short incident waves, we have $\alpha_0 = (\beta H)^{-1/6} + O(\beta^{-1/2})$. Then,

$$|R| = 1 - 2(\beta H)^{-1/6}/\gamma + O(\beta^{-1/2}), \quad T^* = 2\beta^{-1/3} H^{1/6} \sqrt{K_1'(\gamma)/(6\gamma)} + O(\beta^{-1/2}).$$

For $\beta \to 0$, i.e., for a very flexible plate or long incident waves, $\alpha_0 = \gamma - \beta\gamma^4/K_1'(\gamma) + O(\beta^2)$. The coefficients are given by

$$|R| = \beta \frac{\gamma^3}{2K_1'(\gamma)} + O(\beta^2), \qquad T^* = 1 - \beta \frac{6\gamma^3 + 3\gamma^3 H(\gamma^2 + 1)}{2K_1'(\gamma)} + O(\beta^2).$$

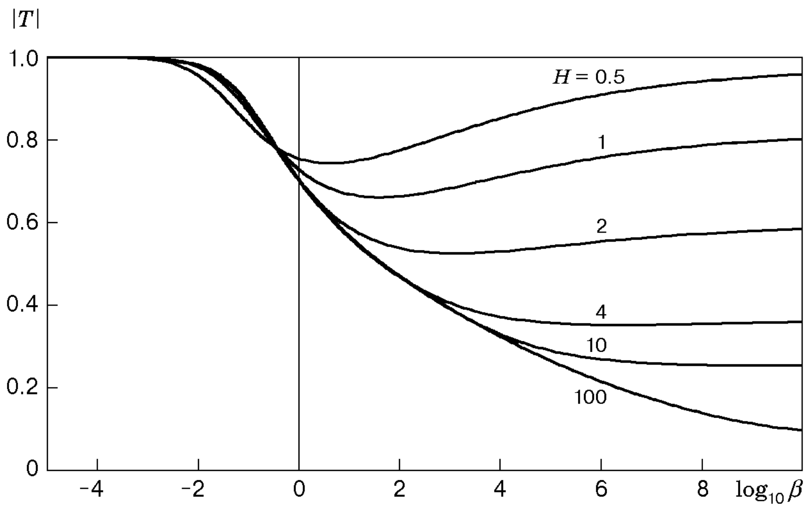

Fig. 5. Amplitude of the potential $|T|$ versus β for various H.

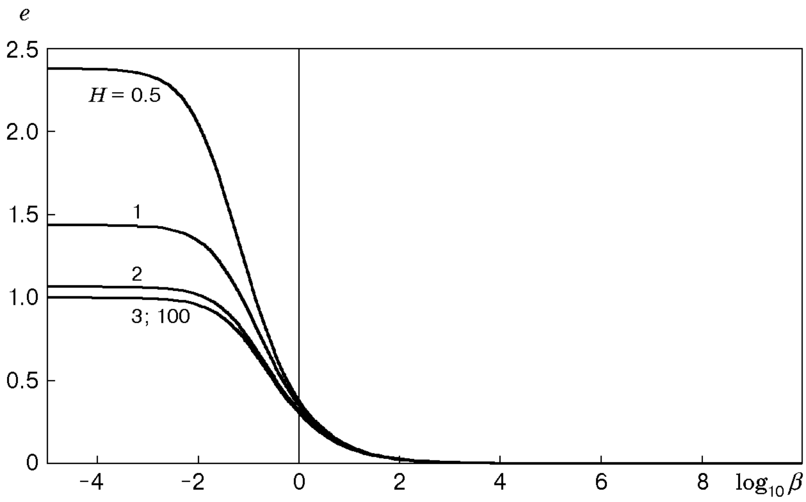

Fig. 6. Nondimensional maximum deflection amplitude e versus β for various H.

For small depth ($H \to 0$), we have $\alpha_0 = (\beta H)^{-1/6} + O(H^{1/2})$. Then,

$$|R| = 1 - 2\gamma(\beta H)^{1/6} + O(H^{1/3}), \qquad T^* = 2\beta^{-1/6} H^{1/3} \sqrt{\gamma K_1'(\gamma)/6} + O(H^{1/2}).$$

At the limit $H \to \infty$, we have the model of an infinitely deep fluid [10].

Numerical Results. The above calculations revealed that the reflection and transmission coefficients depend weakly on the fluid depth but are very sensitive to variation in the parameter β. The dependences of the reflection and transmission coefficients on β are shown in Fig. 4 for $H = 100$ and $H = 0.5$. In the examined range of β, the values of $|R|$ and T^* for $H = 100$ practically coincide with the corresponding values for an infinitely deep fluid [10]. For small values of β, the plate behaves as a thin film and does not affect wave propagation. The reflection coefficient is small, and the transmission coefficient is close to unity. For large values of β, the plate is very rigid, and the reflection coefficient and transmission coefficients tend to unity and zero, respectively.

Curves of the potential amplitude $|T|$ versus β for various H are shown in Fig. 5. The strong dependence on the fluid depth can be clearly seen. Curves of nondimensional maximum strains of the plate in the transmitted wave versus β are shown in Fig. 6 for various fluid depths. As is evident from Fig. 6, for small H and β, the strains of the plate can be appreciable, which is explained by large values of the wavenumber α_0 (see, Fig. 2). Thus, long waves, especially in shallow water, is the most serious hazard to the plate from the viewpoint of structural failure.

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